MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2022/2023

PEM0036 – CALCULUS

(Foundation in Engineering)

10 JULY 2023
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of **SIX** printed pages including the cover page and appendix.

2. Attempt **ALL FOUR** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write all your answers in the answer booklet provided. All necessary working steps MUST be shown.

APPENDIX

BASIC DIFFERENTIATION AND INTEGRATION FORMULAS

$$\frac{d}{dx}[\sin x]=\cos x \qquad \frac{d}{dx}[\cos x]=-\sin x$$

$$\frac{d}{dx}[\tan x]=\sec^2 x \qquad \frac{d}{dx}[\cot x]=-\csc^2 x$$

$$\frac{d}{dx}[\sec x]=\sec x\tan x \qquad \frac{d}{dx}[\csc x]=-\csc x\cot x$$

$$\frac{d}{dx}[e^x]=e^x \qquad \frac{d}{dx}[\ln x]=\frac{1}{x};\quad x>1$$

$$\frac{d}{dx}[\sin^{-1}x]=\frac{1}{\sqrt{1-x^2}};\quad |x|<1 \qquad \frac{d}{dx}[\cos^{-1}x]=-\frac{1}{\sqrt{1-x^2}};\quad |x|<1$$

$$\frac{d}{dx}[\tan^{-1}x]=\frac{1}{1+x^2} \qquad \frac{d}{dx}[\cot^{-1}x]=-\frac{1}{1+x^2}$$

$$\frac{d}{dx}[\sec^{-1}x]=\frac{1}{|x|\sqrt{x^2-1}};\quad |x|>1 \qquad \frac{d}{dx}[\csc^{-1}x]=-\frac{1}{|x|\sqrt{x^2-1}};\quad |x|>1$$

$$Area=\int_a^b [f(x)-g(x)]\,dx$$

$$Volume\ (Disk)=\pi\int_a^b [f(x)]^2\,dx$$

$$Volume\ (Washer)=\pi\int_a^b [f(x)]^2-[g(x)]^2\,dx$$

$$Volume\ (Cylindrical\ Shells)=\int_a^b 2\pi(shell\ radius)(shell\ height)\,dx$$

Continued...

Answer ALL questions [100 marks].

QUESTION 1 [25 marks]

a. Find each limit if it exists.

i. $\lim_{x\to 1.5} (x-1.5)^2 + e^{20}$ [1.5 marks]

ii. $\lim_{x\to 0} \left(\frac{2-2\cos x}{1-\sin x} + \frac{e^x-1}{1-e^x}\right)$ [2.5 marks]

b. If $f(x) = \frac{x^2}{4-x^2}$, determine the following:

i. $\lim_{x\to -\infty} f(x)$. [2 marks]

ii. The horizontal or/and vertical asymptote of the function $f(x)$. [3.5 marks]

iii. Is the function $f(x)$ continuous at $x = 2$? Justify your answer. [2 marks]

c. Given the inequality $\frac{x^2}{2} - \frac{x^4}{24} \leq 1 - \cos x \leq \frac{x^2}{2}$ holds for all values of x.

By using the Sandwich Theorem, find $\lim_{x\to 0} \left(\frac{1-\cos x}{x^2}\right)$. [3.5 marks]

d. A function f is defined by

$$f(x) = \begin{cases} 2e^x + 1 & ; \quad x \leq 0 \\ \frac{2}{x} + 1 & ; \quad 0 < x < 10 \\ k & ; \quad x \geq 10 \end{cases}.$$

i. Find $f(0)$ and $\lim_{x\to 0} f(x)$. [4.5 marks]

ii. State the value(s) of x, where $f(x)$ is discontinuous. [1 mark]

iii. If $\lim_{x\to 10} f(x)$ exist, find the constant value of k. [2.5 marks]

iv. Determine whether $f(x)$ is continuous at $x = 10$? [2 marks]

Continued...

QUESTION 2 [25 marks]

a. Find the derivative of $f(x) = \ln(3ex) - 2(3^x) + e^{-x}$. [2.5 marks]

b. A particle moves along the x-axis with position,

$$s(t) = \int_0^t (3t^2 - 4t + 2)\, dt \quad ; \quad \text{where } t \text{ is measured in seconds.}$$

Find the velocity of the particle at time $t = 2$ seconds. [2.5 marks]

c. If $y = e^{-\sin 2x}$, show that $\frac{d^2y}{dx^2} + (2\cos 2x)\frac{dy}{dx} - (4y \sin 2x) = 0$. [7 marks]

d. For the function $f(x) = 2x^3 - 3x^2 - 12x + 7$, find the following:

i. Find the point(s) on the curve $f(x)$ where the tangent line is parallel to the line $y = 24x - 1$. [6 marks]

ii. Intervals on which the function is $f(x)$ increasing or decreasing. [5 marks]

iii. Extreme values(s) and where they occur. [2 marks]

Continued ...

QUESTION 3 [25 marks]

a. i. $\int \frac{1}{x+1} - \frac{1}{1-x} - \frac{e^x}{e^x-1}\, dx.$ [2.5 marks]

ii. Evaluate $\int_0^1 \left(\frac{1-e^x}{e^x}\right) dx.$ [2.5 marks]

b. Suppose f and g are continuous that $\int_0^4 f(x)\,dx = 3$, $\int_1^4 f(x)\,dx = -5$, and $\int_0^1 g(x)\,dx = 2$. Find $\int_0^1 2f(x) - g(x)\,dx$.. [4 marks]

c. A curve $y = \sqrt{x-1}$ and a straight line are shown in the Figure 1 below.

Given that the straight line has x-intercept and y-intercept at 1 and $-\frac{1}{2}$ respectively. The straight line meets the curve at (1, 0) and A (k, 2).

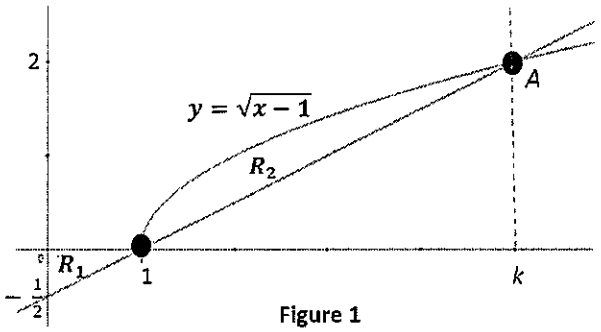

Figure 1

i. Find the value of k. [2 marks]

ii. Find the equation of the straight line. [3.5 marks]

iii. Evaluate $\int_1^5 \sqrt{x-1}\ dx$. [2.5 marks]

iv. Find the area of the region R_2 that bounded by the straight line and the curve. [3 marks]

v. Calculate the generated volume when the region R_1 which is bounded by the straight line, the x-axis and the y-axis is revolved completely about the line $x = 0$. [5 marks]

Continued ...

QUESTION 4 [25 marks]

a. Find the particular solution to the differential equation $\frac{dy}{dx} = 2x^2 + x + \pi$ with initial condition $y(0) = 2$. [3 marks]

b. Solve the following second-order constant-coefficient non-homogeneous ODE:

$y'' + 4y' + 3y = 3e^{-4x}$. Verify your answer. [12 marks]

c. Suppose the production of vegetables in a farm affected by pests using an organic pesticide can be modelled by a non-linear differential equation:

$$t\frac{dx}{dt} + x + t(x + k) = 0\,,$$

where x is the production over time in kg per week, t is the time in week and k is a constant.

i. Show that the substitution $y = xt$ transforms the non-linear differential equation to a linear differential equation $\frac{dy}{dt} + y = -kt$. [3.5 marks]

ii. Solve the linear differential equation in Q4c(i):

$\frac{dy}{dt} + y = -kt$. [6.5 marks]

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